practice

How would you find:

- $$\int \frac{\mathrm{d}x}{\sin(x)\ \cos^2(x)}$$
- $$\int \tan^4(x)\,\mathrm{d}x$$
- $$\int \frac{\cos(x)\,\mathrm{d}x}{2\sin^2(x)+\sin(x)-1}$$

1. Turn into sec*csc^2, use csc^2=1+cot^2, multiply through...
   Answer = ln |sec x + tan x| - csc x + C
2. Split as tan^4=tan^2 * (sec^2-1), split into two integrals, use u=tan x for the first, tan^2=sec^2-1 for the second
   Answer = 1/3 * tan^3 (x) - tan(x) + x + C
   (Stewart 7.2.24)
3. Let u=sin(x), then do partial fractions.
   Answer = 1/3 * ln | (2 sin(x)-1) / (sin(x) + 1) | + C

tuesday, 24 november:
course evaluations
§ 9.3 - separable equations
§ 9.4 - exponential growth and decay

thursday, 26 november:
no school (thanksgiving)

tuesday, 1 december:
review for final
quiz v: §§ 9.1, 9.3

thursday, 3 december:
review for final
homework viii due: 9.1.4, 9.1.12, 9.3.12, 9.3.36, 9.4.4, 9.4.14.

friday, 4 december:
mslc: webwork workshop @ 12:30, 1:30, 2:30, 3:30, 4:30 in SEL 040
webwork homework vii due @ 11:55 pm

monday, 7 december:
extra credit project 3 due @ 6 am
mslc: final exam review @ 7:30 pm in HI 131

tuesday, 8 december:
final exam @ 5:30 pm

separable equations

A separable equation is a first-order differential equation in which the expression for $\mathrm{d}y/\mathrm{d}x$ can be factored as a function of x times a function of y. In other words, it can be written in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)\,g(y)$$

This definition is verbatim from Stewart's section 9.3.

Solve by multiplying both sides by dx, dividing by g(y) and integrating.

example

Solve:

$$\frac{\mathrm{d}z}{\mathrm{d}t} + e^{t+z} = 0$$

Split up e^(t+z), separate. Answer:

z = - ln (e^t - C)

(Stewart 9.3.10)

example

Solve:

$$\frac{\mathrm{d}u}{\mathrm{d}r} = \frac{1+\sqrt{r}}{1+\sqrt{u}}$$

In the last example, we could explicitly solve the equation. Here we can't.

u + 2/3 u^(3/2) = r + 2/3 r^(3/2) + C

(Stewart 9.3.6)

initial value problems

An initial value problem is an ordinary differential equation together with a specified value, called an initial condition, of the unknown function at a given point in the domain of the solution.

Definition from Wikipedia, accessed 24 November 2009.

example

Find an equation of the curve that satisfies

$$y' = 4\,x^3\,y$$

and whose y-intercept is 7.

This is an intial value problem with initial condition y(0)=7.

Answer:
y=7 e^(x^4)

(Stewart 9.3.19)

exponential growth

If $y(t)$ is the value of a quantity y at time t and if the rate of change of y with respect to t is proportional to its size $y(t)$ at any time, then

$$\frac{\mathrm{d}y}{\mathrm{d}t} = k\,y$$

The law of natural growth applies to compound interest and population growth (in certain circumstances). Natural decay applies to radioactive decay, decay of biological compounds (in certain cases), etc...

If $k>0$, we call this the law of natural growth;
if $k<0$, we call this the law of natural decay.

exponential growth

We saw last time that the equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = k\,y$$

has solution

$$y(t) = y_0\,e^{k\,t}$$

where $y_0 = y(0)$.

If we forgot, we can check two ways:

1. just plug in.
2. this is a separable equation. solve it.

example

^{14}C has a half-life of about 5730 years. A piece of parchment was discovered that had about 74% as much ^{14}C radioactivity as does plant material today. Estimate the age of the parchment.

k=ln(2)/5730

t=-5730 (ln .74)/ln 2
approx 2489 years

(Stewart 9.4.11)

compound interest

If an amount A_0 is invested at an interest rate r which is compounded n times a year, then after t years, the value of the investment is

$$A_0 \left(1 + \frac{r}{n}\right)^{n\,t}$$

In each compounding period, the interest rate is r/n, total of nt compounding periods in t years.

(Definition from Stewart)

compound interest

Letting $n \to \infty$ we arrive at the formula for continuously compounded interest:

$$\begin{aligned}\lim_{n\to\infty} A_0 \left(1 + \frac{r}{n}\right)^{n\,t} &= \lim_{n\to\infty} A_0 \left[\left(1 + \frac{r}{n}\right)^{n/r}\right]^{r\,t} \\ &= A_0 \left[\lim_{n\to\infty} \left(1 + \frac{r}{n}\right)^{n/r}\right]^{r\,t} \\ &= A_0 \left[\lim_{m\to\infty} \left(1 + \frac{1}{m}\right)^{m}\right]^{r\,t} \\ &= A_0\, e^{r\,t}\end{aligned}$$

Here m=n/r.

example

How long will it take an investment to double in value if the interest rate is 6% compounded continuously?

t=50/3 * ln 2
(about 11.55 years)

(Stewart 9.4.20a)

coming soon

- quiz v (§§ 9.1, 9.3) next tuesday
- extra credit project 3 due on 7 december